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PROBABILISTIC NEURAL NETWORKS FOR SEISMIC DAMAGE MECHANISMS PREDICTION

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SUMMARY

The procedure commonly employed to assess the seismic vulnerability of buildings uses simplified qualitative and quantitative observations obtained from the measured data entered into report forms. In Italy, the data sheets adopted by the National Defence Group against Earthquakes (Gruppo Nazionale per la Difesa dai Terremoti—GNDT) play a unifying and reference role. This paper proposes a method for the processing of the data contained in such report forms which is based on probabilistic neural networks producing a Bayesian classification. The final goal is to exploit the fundamental learning and generalization capabilities of neural networks to obtain an estimate of the vulnerability of structural systems. In particular, the aim is to be able to predict the damage mechanisms which may be triggered in the macro-elements of public worship buildings. Copyright © 1999 John Wiley & Sons Ltd.

KEY WORDS: Bayesian classification; probabilistic network; structural macro-elements; damage mechanisms; vulnerability

1. INTRODUCTION

Italy has a wealth of buildings that have been erected over many centuries, many of which survived by withstanding the offences of time, deterioration, war and natural disasters, or were heavily damaged and then repaired as precious mementoes of the national cultural identity. Our country has been hit by earthquakes in a diffused and repeated manner, albeit not as violently as other regions of the earth. Among the buildings of greatest historical and cultural interest, we should mention churches containing architectural and figurative art treasures whose protection is of pre-eminent importance for the preservation of the cultural identity of the nation and humankind.[1,2] From past experience we know that churches are often more vulnerable than ordinary buildings when exposed to seismic events.[3] The number of catholic churches of historical and artistic relevance is great and it poses the problem of making available to the municipal authorities reliable and easy to use tools for the prediction of structural damage, and to define effective strategies of intervention, improvement and protection.[1–3]

In 1989 an interesting investigation[4] explored the vulnerability and damage mechanisms observed in large number of churches of Friuli (a north-eastern Italian region that was hit by

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a violent earthquake, Richter 6·8, in May 1976). Since churches are atypical buildings, of unique design, and are made of major structural components with somewhat recurring features, the notion that inspired the investigation was to break down each church into basic components, referred to as 'macro-elements' (facade, apse, side walls, spire, etc.). Each of the macro-elements was examined to recognize the possible mechanisms of damage formation and to establish a correlation between the latter and the macro-elements' type and dimensional characteristics. If the databases available are comprehensive enough, this makes it possible to identify several type classes and as many classes of damage mechanisms, mutually correlated by statistically defined relationships.

In this paper, in accordance with the guidelines described above, the relationship between damage mechanisms and macro-elements is explored by resorting to an automated procedure which uses probabilistic neural networks[5] and, in particular, a network proposed by Comon[6] (a variant of the better known network developed by Specht[7,8]), which proves very useful as well as an easy to use and intrinsically robust tool.

Artificial neural networks are data processing systems possessing learning and generalization capabilities. In a supervised neural network, training consists of presenting a set of examples and letting the network build up, on the basis of a well-defined algorithm, the interior structure it needs to perform its intended task.[9] In mathematical terms this amounts to supplying a set of vector pairs, i.e. an input vector (X) and an output vector (Z), and having the network associate all the $X - Z$ pairs, with the least possible error, so as to build up a unique functional relationship $Z = F(X)$, no matter how complex is the phenomenon to be interpreted. Having concluded the training stage, the network must be able to effectively predict the output vector Z most closely associated with the input vector X. In classical neural networks, the answer is univocal, and vector Z is the one involving the greatest possibility of occurrence. Probabilistic networks, on the other hand, supply several answers Z_i, each of which is associated with an estimate of the respective probability.[10]

In the case being considered, the probabilistic network yields a Bayesian classification;[11] we want to work out an estimate of the likelihood of each damage mechanism occurring in a structure whose type and geometrical attributes are known. The construction of the probabilistic network is quite simple, but the delicate and critical part of its utilisation is the way in which the qualitative and quantitative data collected are translated into the vectors of conventional numerical codes. The information contained in '*in-situ*' measurement data sheets was rearranged, elaborated and schematised so as to reduce to the greatest extent possible any causes of ambiguity and uncertainty of attribution.

In the following section the original proposals formulated by the authors are backed up by a detailed description of the general methodology of probabilistic Bayesian network, taken from References [6–8], to assist the reader in gaining an understanding of the problems in question.

2. BAYESIAN CLASSIFICATION

It is well known that Bayes formulated his theorem on the basis of binomial distributions and that the applicability of the theorem was then generalized to a wide set of statistical distributions in the discrete, as well as continuous, random variables domain. We can start by summarizing the formulation of Bayes' theorem for discrete distributions.

Given a space S, let us consider a partition consisting of mutually exclusive ($A_i \cap A_j = 0$, $i \neq j$) and exhaustive ($S = A_1 \cup A_2 \cdots \cup A_n$) events $A_1, A_2, \ldots, A_n$ and an arbitrary event B; since this event may occur together with A_1 or $A_2, \ldots, A_n$, the probability of the occurrence of event B is given by

$$P(B) = P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n) = \sum_{j=1}^{n} P(B|A_j)P(A_j) \tag{1}$$

This formulation, known as 'absolute probability theorem', retains its validity even in the presence of an infinite number of events. A direct consequence of the conditional and absolute probability theorem is Bayes' theorem:[12]

$$P(A_i|B) = [P(B|A_i)P(A_i)] \Big/ \left[\sum_{j=1}^{n} P(B|A_j)P(A_j) \right] \tag{2}$$

Bayes' theorem is also known as the theorem of the probability of the causes, in that it makes it possible to determine the probability that events A_i were the cause of the occurrence of event B. Bayes' theorem is used to convert an '*a priori*' probability estimate $P(A_i)$ into an '*a posteriori*' probability estimate $P(A_i|B)$ knowing the conditional probability $P(B|A_i)$. This theorem enables one to determine the conditional probability of event A_i given that event B has occurred.

Let us assume that the random event B is the value taken by a vector of continuous variables, intended as a set of n measures, and that we have to solve a classification problem. Comon suggests the following strategy:[6] the classification of a set S into K different classes leads to the subdivision of S into K domains D_i whereby all the vectors falling in D_i are assigned to class ω_i. In the Bayesian context, these domains can be estimated thanks to the minimisation of a *risk function* R, with the aim of penalising, through the definition of a *loss function* $L(i, j)$, the classification in class ω_i of an element X which belongs to class ω_j (the losses associated a correct classification $L(i, j = i)$ are taken to be equal to zero).[8] The risk function is defined in terms of the expected value of the losses over the set of observations mixed in all the classes:

$$R = \sum_{i,j=1}^{K} L(i,j)P(\omega_j) \int_{D_i} p(u/\omega_j)\,\mathrm{d}u \tag{3}$$

where $P(\omega_j)$ is the *a priori probability* of an observation X originating from class ω_j, and $p(u|\omega_j)$ is the probability density function that the observation originating from class ω_j takes on the value of u. Comon shows that the assumption of exhaustiveness and mutually exclusiveness leads to simplified version of the previous formula: the correct class is identified by the highest value of the integral contained in equation (3). The main limit of such approach is that it can only extract the best choice, in probabilistic terms, but it is unable to estimate the probability of belonging to each class.

For our purposes, we shall follow an alternative strategy, obtained directly through the application of Bayes' theorem specifically oriented to the estimate of the probability of X belonging to each class.[11]

In the vulnerability context, we want to classify a set of m-dimensional vectors $X = \{x_1, x_2, \ldots, x_m\}^{\mathrm{T}}$ representing the typological characteristics of a building, into a discrete number of classes $\omega_1, \omega_2, \ldots, \omega_K$ representing the different damage mechanisms. In this case (discrete

parameter) Bayes' theorem asserts that the probability mass function of ω_i for a given vector X is:

$$h(\omega_i|X) = [p(X|\omega_i)g(\omega_i)] \Big/ \left[\sum_{j=1}^{K} p(X|\omega_j)g(\omega_j) \right] \tag{4}$$

where:

- $g(\omega_i)$ denotes the '*a priori*' probability density since it is determined prior to observing vector X in the current experiment (based upon previous understanding)
- $p(X|\omega_i)$ denotes the probability density function of X, conditional upon the class ω_i
- $h(\omega_i|X)$ is called the '*a posteriori*' probability mass function of ω_i, given X, since it is determined after observing the current set of data.

Equation (4) estimates the '*a posteriori*' probability density that vector X belongs to class ω_i. The key to using Bayes' theorem in classification problems is the ability to estimate $p(X|\omega_i)$. In this context we adopt uniform distributions for the '*a priori*' probability densities and have to estimate the different $p(X|\omega_i)$ by means of a probabilistic network based on training patterns because all that is given is a set of training samples (m-dimensional vectors Y obtained from vulnerability data sheets) which provide the only clue to the unknown probability densities:

$$\begin{array}{lllll} Y_1^{(1)} & Y_2^{(1)} & , \dots , & Y_{N1}^{(1)} : \in \omega_1 \\ \vdots & \vdots & & \vdots \\ Y_1^{(K)} & Y_2^{(K)} & , \dots , & Y_{NK}^{(K)} : \in \omega_K \end{array} \tag{5}$$

3. THE PROBABILISTIC NETWORK

The estimate of the probability density is obtained through a non-parametric approach as, in the classification of vulnerability data sheets, there is no specific parametric relationship between the components of each vector and the damage classes. Among the different non-parametric approaches available, we have used the core estimators technique, which supplies continuous estimates at all times and is ideally suited to deal with small sized samples.[8] The estimate $p(u)$ of a density $p(u)$ based on the observation of N samples Y_i is given by the following expression:

$$\hat{p}(u) = \frac{1}{N} \sum_{n=1}^{N} h_\alpha(u - Y_i) \tag{6}$$

where $h_\alpha(u)$ is a kernel function defined as *probability core*. Index α shows that the core function depends on a parameter which depends on N to provide the estimator with suitable asymptotic properties. The probability core function used is

$$h_\alpha(u - Y_i) = C(m, \alpha)\,[1 + \exp(\alpha \|u - Y_i\|^2/2)]^{-1} \tag{7}$$

where $C(m, \alpha)$ is a normalization constant depending on dimension m of the training vectors and on parameter α. If the parameter $\alpha = \alpha(N)$ is selected so that

$$\lim_{N \to \infty} \alpha = \infty \quad \text{and} \quad \lim_{N \to \infty} \frac{N}{\alpha} = \infty \tag{8}$$

then it can be demonstrated that the estimator $\hat{p}(u)$ simply converges via quadratic means towards $p(u)$ at all points where the latter is continuous.[8] A convenient choice can be $\alpha = \sqrt{N}$, where N is the number of samples available.

3.1. Network architecture

As mentioned before, each macro-element is represented by a vector X containing construction typologies and geometrical data of the building and is associated (according to equation (5)) with the class ω_i representing the specific damage mechanism suffered by the building itself.

The complete network producing a classification into K classes of damage is made up of K identical subnetworks (Figure 1) in which training and classification stages are conducted at different times. The supervised training of each of these subnetworks (each of which therefore represents a specific damage formation mechanism) is performed separately by selecting the network corresponding to the class being observed.

3.2. Subnetwork training

The domain D_i of m-dimensional vectors $Y_1^{(i)}, Y_2^{(i)}, \ldots, Y_{Ni}^{(i)}$ belonging to the class ω_i is represented by P nodes or neurons each of which is characterized by an m-dimensional vector $W_{p,i}$ (with $p = 1, \ldots, P$), referred to as centroid later on because it represents the barycentre of the node, and by a scalar coefficient $a_{p,i}$ which measures the degree of activation of the node itself (Figure 2).

The training stage begins by arranging at each node of the subnetwork a vector $W_{p,i}$ while the coefficients $a_{p,i}$ are taken to be 1. At first the centroids $W_{p,i}$ are defined through a random generation procedure or by using a number P of training vectors clearly representing the type class.

At the start of a subnetwork i (Figure 2) we find the so-called squaring cell; for each training vector Y_i of size m, that is presented at the input, a vector $\bar{Y}_i$ of size $m+2$ is given at the output, defined by:

$$\bar{Y}_i = [\| Y_i \|^2 \, Y_i - 1/2]^{\mathrm{T}} \tag{9}$$

Then this vector is distributed to the P neural cells where the same transformation is made for the each of the centroids:

$$\bar{W}_{p,i} = [-1/2 W_{p,i} \, \| W_{p,i} \|]^{\mathrm{T}} \tag{10}$$

The previous equations (9) and (10) enable us to find the Euclidean distance of Y_i from each centroid as

$$\bar{W}_{p,i}^{\mathrm{T}} \bar{Y}_i = -\| Y_i - W_{p,i} \|^2 / 2 \tag{11}$$

This quantity is assigned to each of the P neurons belonging to the subnetwork, which, in their turn, through the transfer function defined by the sigmoid function, supply at the output the quantities:

$$S_{p,i} = f_\alpha(-\| Y_i - W_{p,i} \|^2/2) = \frac{1}{1 + \exp^{\alpha \| Y_i - W_{p,i} \|^2/2}} \tag{12}$$

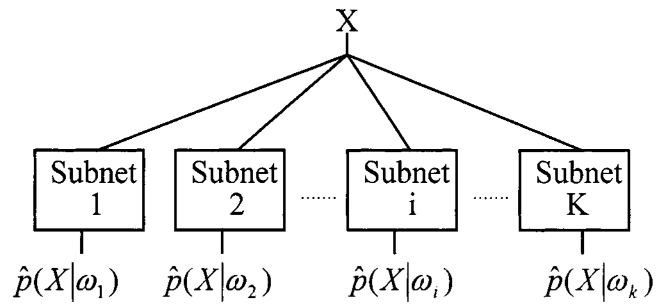

Figure 1. Network architecture

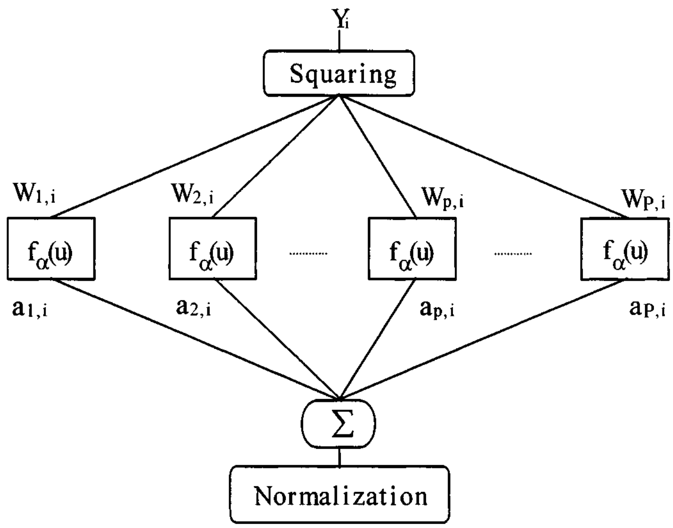

Figure 2. Subnetwork architecture

The vector Y_i is assigned to the neuron $p_{0,i}$ which produces the maximum answer $S_{p,i}$ and the new centroid vector $W_{p0,i}^{(\text{new})}$ and the coefficient $a_{p0,i}^{(\text{new})}$, updated, become

$$W_{p0,i}^{(\text{new})} = \frac{a_{p0,i} W_{p0,i} + Y_i}{a_{p0,i} + 1}, \quad a_{p0,i}^{(\text{new})} = a_{p0,i} + 1 \tag{13}$$

In this manner, whenever a training vector is assigned to one of the P neurons, its activation coefficient increases and its centroid vector is replaced, according to equation (13), by adding to the older set the new training vector Y_i.

The automatic grouping of all the training vectors belonging to a specific subnetwork is achieved according to the rule of evolution of the centroids and can be improved by using the same training vectors a number of times (iterations) in a different order of presentation. The

learning stage is considered completed when the evolution of the centroids (that is the variation of their position) is no longer significant. The number of iterations, depending on the number of neurons adopted and on the choice of the initial centroids, may be very high; in that case it is worthwhile to limit the computer memory requirements by inserting a real number η, selected in the]0,1] range, in equation (13) for the activation coefficients:

$$a_{p0,i}^{(\mathrm{new})} = \eta(a_{p0} + 1) \tag{14}$$

3.3. *Classification stage*

At the classification stage the centroids $W_{p,i}$ are fixed. The outputs of the P neurons, for a given observation vector X, are weighted by the $a_{p,i}$ coefficients and then are added up as shown in Figure 2. At this point a normalization procedure is applied in order to ensure that the estimate actually corresponds to a probability density. The expression of the estimated probability density of a vector X conditional upon the class ω_i is

$$p(X|\omega_i) = \frac{1}{A(i)} \sum_{p=1}^{P} a_{p,i} f_\alpha(-\|X - W_{p,i}\|^2/2) \quad \text{where } A(i) = \sum_{p=1}^{P} a_{p,i} \tag{15}$$

The normalization factor $A(i)$ takes into account the actual totals present for the construction of the density relating to class ω_i.

This procedure, repeated for all the K classes (Figure 1) allows us to classify an observation vector X, representing a given building, into one of the K subnetworks representing the different damage classes.

4. MONUMENTAL BUILDINGS

The sample of churches considered in this paper comes from the census taken by CNR–GNDT for about 100 churches damaged by the earthquake that hit Emilia Romagna in 1987 and about 250 churches damaged by the Friuli earthquake in 1976. The data are collected in forms consisting of a first part, the so-called preliminary data sheet, collecting the general information on the church, and a second part including all the specific data sheets (main body of the church, apse, transept, spire, vertical structures, finishing and cladding, damage).[4] The church data sheet also includes a graphic section concerning the geometry of the church itself and the lesions produced by the earthquake.

To characterize a church building from the structural viewpoint and analyse the damage it has suffered, we have used the notions of macro-element and damage. For each macro-element, elementary structural types are identified so that complex types can be described through the sum of the elementary ones. For the facade, for instance, the elementary types are grouped in Figure 3.

The damage mechanism represents a schematic reconstruction of the movements of parts of the macro-elements and the relative displacements deduced from the cracking patterns observed in the churches included in the sample.[11] A mechanism is rated as unitary or simple when the entire damage process is associated with the evolution of a single mechanism, and conversely it is rated as composite if it consists of a main mechanism, affecting the structure as a whole, plus one or more secondary mechanisms, affecting limited portions

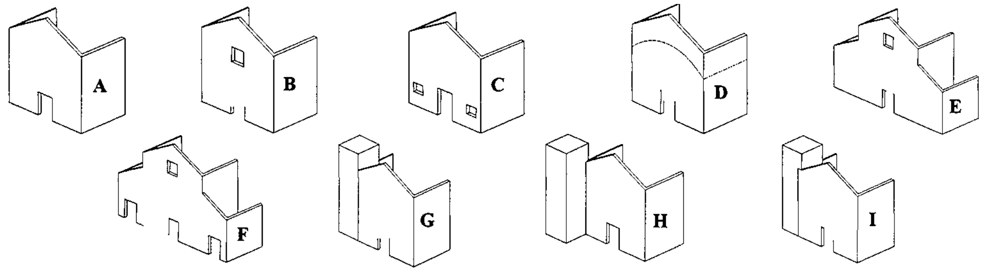

Figure 3. Elementary structural types for the facade macro-element

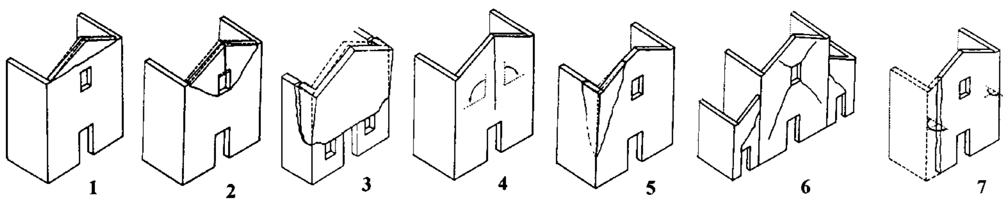

Figure 4. Elementary damage mechanisms for the facade macro-element

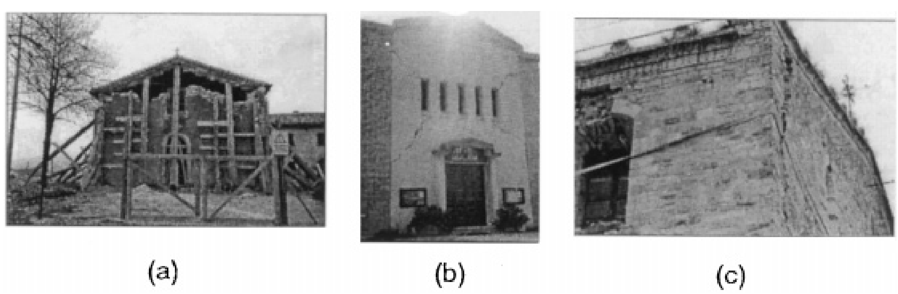

Figure 5. Real images of damage mechanism: (a) mechanism 1; (b) mechanism 6; (c) mechanism 7

of the macro-element. The elementary damage mechanisms identified for the facade are shown in Figure 4.

In Figure 5 some real images are associated with elementary damage mechanism.

4.1. Structure of the training samples

Getting hold of the church data sheets, breaking down the building into macro-elements and interpreting the damage in terms of damage mechanisms are but the first step in the construction of the training vectors. The fundamental step in fact is the choice of the parameters to be introduced into the vector for the training stage. The most important parameters affecting the behaviour of a church building are: metric and type data, structural seismic chronology,

morphological data of the site, type of vertical and horizontal structures (roofing and vaults), presence and type of spire, connections between the various structural elements, presence of annexed bodies (porches, sacristies, buildings), foundations and quality of materials. The parameters taken into account in defining the training vectors are listed in Table I.

The alpha numerical data must be converted into purely numerical data to be processed by the computation program. Furthermore, many of the parameters expressed in numerical form in the church data sheet, such as the parameters expressing maximum church dimensions, have a potentially infinite field of variation. It is therefore necessary to convert such variation fields into discrete ranges of numerical values. These conversions are illustrated in Tables II–IV.

Table II

The maximum width of the church must be measured in the orthogonal direction to the nave while the maximum length of the church must be measured in the direction of the nave. The

Table I. Type parameters

1	Maximum width of the church
2	Maximum length of the church
3	Maximum height of the church
4	Total plan area of the church
5	Gross volume of all constituent elements of the building
6	Structural type of main body
7	Structural type of apse expressed in terms of macro-element
8	Structural type of transept
9	Structural type of spire
10	Types of other buildings annexes to the church (sacristies, porches, etc.)
11	Structural type of roofing
12	Structural type of facade expressed in terms of macro-element
13	Structural type of right side wall expressed in terms of macro-element
14	Structural type of left side wall expressed in terms of macro-element
15	Structural type of triumphal arch expressed in terms of macro-element

Table II. Geometrical parameters

	Variation range	Conversion value		Variation range	Conversion value
	$7 \leqslant L \leqslant 12$	1			
Maximum church width	$12 \leqslant L \leqslant 18$	2		$S < 100$	1
	$18 \leqslant L \leqslant 24$	3	Total plan area of the church	$100 \leqslant S \leqslant 200$	2
	$24 \leqslant L \leqslant 29$	4		$200 \leqslant S \leqslant 400$	3
	$7 \leqslant L \leqslant 15$	1		$400 \leqslant S \leqslant 500$	4
Maximum church length	$15 \leqslant L \leqslant 25$	2		$500 \leqslant S \leqslant 1000$	5
	$25 \leqslant L \leqslant 35$	3	Gross volume of all constituent elements of the building	$V < 1000$	1
	$35 \leqslant L \leqslant 55$	4		$1000 \leqslant V \leqslant 3000$	2
	$5 \leqslant H \leqslant 9$	1		$3000 \leqslant V \leqslant 5000$	3
Maximum church height	$9 \leqslant H \leqslant 12$	2		$5000 \leqslant V \leqslant 9000$	4
	$12 \leqslant H \leqslant 18$	3		$9000 \leqslant V \leqslant 16900$	5
	$18 \leqslant H \leqslant 24$	4			

Table III. Structural parameters

	Description	Church data sheet code	Conversion value
Structural types of main body	Single nave	U	1
	Two-nave body	2N	2
	One nave two aisle body	3N	3
	Body with several aisles	nN	4
	Single nave with chapels	UCP	5
	One-nave two-aisle body with chapels	3NCP	6
	None	I	0
Structural types of apse	Rectangular	RE	1
	Polygonal	PL	2
	Circular	CL	3
	Irregular shapes	CO	4
Type of transept	None		0
	Single nave	U	1
	None		0
Structural types of spire	Adjacent	AD	1
	Included	CM	2
	Separate	ST	3
	None		0
Type of annexes	Sacristy	SA	1
	Porches	PO	2
	Buildings	ED	3
	Sacristy with porches	SAPO	4
	Sacristy with buildings	SAED	5
Structural types of roofing	Information not available		0
	Thrusting type of roofing	A	1
	Non-thrusting type of roofing	B	2
	Potentially thrusting type of roofing	C	3

Table IV. Macro-element parameters

	Type of macro-element	Conversion value		Type of macro-element	Conversion value
				A	1
	A1	1		B	2
	A2	2	Structural types of facade	C	3
	A3	3		F	4
	B	4		B–C	5
Structural types of right and left side walls	C1	5		A-I\C-I\ B-H\B-G	6
	C2	6		C-D\B-E	7
	C3	7		A	1
	A1–B	8		B	2
	A3–D	9	Structural types of triumphal arch	C	3
	B–C2	10		D	4
	B–C2–D	11		A–E	5
	C1–D	12		B–E	6
				D–E	7

maximum height of the church must be determined by taking into account the maximum height reached by the roof top excluding the spire. Total plan area refers to the church proper, leaving out the spires, if separate, and the other annexed bodies, if any. Gross volume (in m^3) refers to the elements of the building, other than the spires, if separate, and the annexed bodies, if any.

Table III

The information reflecting the structural type of the apse is introduced with reference to the type classes of the apse macro-element; in addition to the three type classes (described in Figure 4(a)) it is also necessary to take into account an additional class for shapes other than the regular ones.

The information reflecting type of spire has not been introduced on the basis of the macro-element theory; the damage suffered by the spire cannot be investigated, because of poor information, and we have only tried to assess the influence of the spire on the damage to the facade and lateral walls. Hence, we have only introduced the information concerning its relative position to the main body of the building. The term 'bodies annexed to the church' refers to the presence of porches, sacristies and veritable buildings which are not part of the church itself but interact with it.

Table IV

The structural type of the facade (Figure 3), triumphal arch (Figure 6(b)) and lateral walls (Figure 6(c)), expressed in terms of macro-element, are given by the elementary types into which the macro-element has been broken down. To identify the classes in which to subdivide these

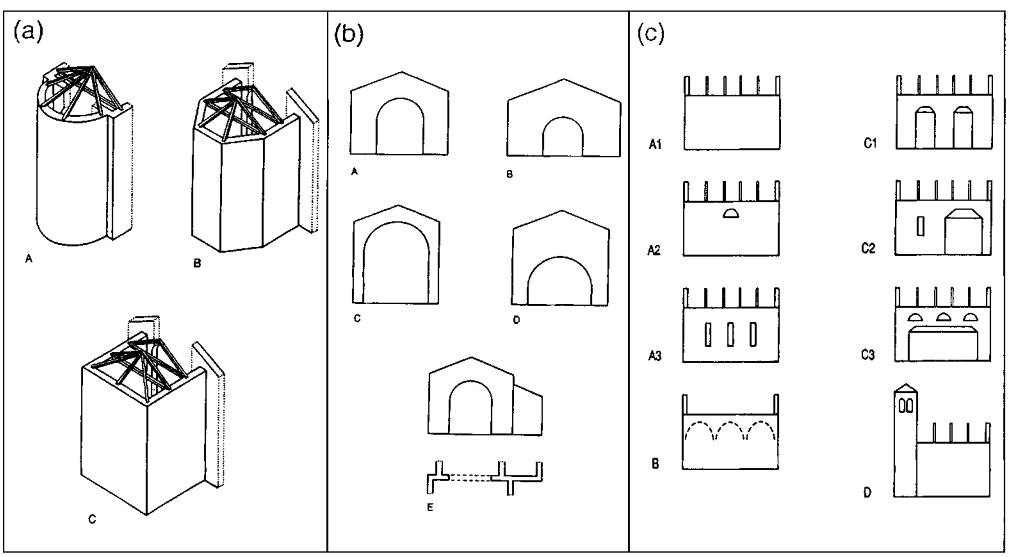

Figure 6. (a) Types of apse macro-element; (b) Types of triumphal arch; (c) Types of side wall macro-element

parameters, it is therefore necessary to examine the combinations of elementary types that appear in the sample being considered.

Once converted into the codes listed above, the parameters contained in the church data sheets were subjected to a normalization process so as to optimize the workings of the probabilistic network.

5. TRAINING, CLASSIFICATION AND RESULTS

5.1. Building up the training vectors

On the basis of the criteria illustrated above, a training sample has been built, containing in sequential form the vectors made up of the 15 descriptive parameters of each individual church building. The approach adopted in this investigation makes it possible to analyse separately the damage that can be inflicted on each individual macro-element. A network consisting of as many subnetworks as the damage mechanisms observed is constructed for each macro-element. In the case being considered, seven different damage mechanisms have been identified for the facade and the side wall macro-elements, which will be matched, in the terminology of neural networks, by seven different subnetworks. Each subnetwork was made up of 10 nodes or neurons.

The training sample is organized according to equation (5) in seven classes associated with the seven different damage mechanisms. Each class contains the vectors Y relating to the churches in which the corresponding damage mechanism has been observed. If a single macro-element displays several damage mechanisms at play simultaneously, the correspondent training vector is introduced into all the damage classes involved. For instance, if the macro-element being considered reveals the workings of damage mechanisms 3 and 5, its training vector is introduced into both the third and the fourth class.

5.2. Training

The training sample refers to about 250 buildings (fewer than the buildings included in the census, because some of the data sheets did not contain sufficient data) and the correspondent vectors are subdivided in a nearly uniform manner into the seven classes of damage for the macro-element facade. The most efficient results were obtained by using 10 neurons for each subnetwork with coefficient $\eta = 0{\cdot}5$. The efficiency of the training process was tested at different levels of learning (that is, at different number of iterations). The training stage was seen completed at 180 000 iterations because further training did not change the positions of the centroids.

5.3. Classification

The final aim of this study is to obtain a network which, at the end of the learning stage, is able to supply a prediction of damage mechanisms for each single monumental building. During the classification stage, the network receives input vectors X consisting of the representative parameters of monumental buildings not included in the training sample; as a result, on the basis of the decision making criteria acquired at the training stage, the network supplies a prediction of the damage mechanisms that are most likely to occur.

The reliability of the predictions obtained depends on the quality of the training data and is evaluated in successive stages through validation operations. A validation operation consists of

presenting the network with examples (10 buildings in this case) that have been previously excluded from the training sample and whose damage mechanisms are known, and of verifying the network's ability to correlate the highest probability value with the damage mechanisms actually observed.

5.4. Results

The applicability of probabilistic networks to churches through the method described above was evaluated by comparing the predictions formulated by the network with the data concerning the damage actually caused by earthquakes in a number of churches previously excluded from the training data sets to be able to conduct the validation tests. Collapse mechanisms were correctly predicted for all structural macro-elements. For each one of them the outcome shows a satisfactory degree of stability over the 180 000th iteration. As an example, let us examine the outcome of the predictions concerning the facades of a number of churches rising in different areas.

5.4.1. Church of S. Chiara in Venzone

The earthquake triggered damage mechanisms Nos. 2 (Out of plane rotation following the formation of a cylindrical hinge with slanted axes) and 3 (Out of plane rotation with the formation of a cylindrical hinge with horizontal axis). From Figure 7 it can be seen that initially (after 18 000 iterations) only mechanism 3 was recognized, whilst at a later stage in the training session (180 000 iterations), the probability value assigned to mechanism 2 also increased.

5.4.2. Church of S. Maria Assunta in Scandiano

The damage mechanism actually triggered by the quake was No. 4 (Translation in the plane of the facade). Figure 8 shows that the network identified the wrong mechanism (No. 7) at first, and then, with increasing number of iterations, pinned down the correct mechanism (No. 4).

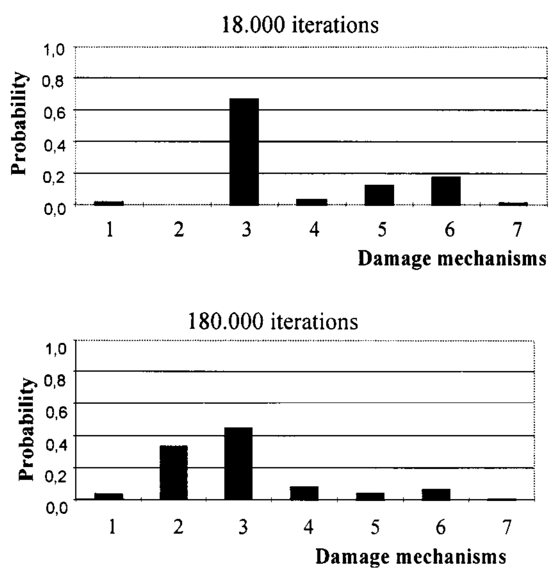

Figure 7. Prediction of damage mechanisms affecting the church of S. Chiara in Venzone

5.4.3. Church of S. Chiara in Carpi

On the facade macro-element, we should identify the activation of damage mechanism No. 4 (Translation in the plane of the facade). The wrong mechanism (No. 7) is identified at first, and then, with increasing number of variations, the probability value ascribed by the network to mechanism No. 4 is seen to increase and eventually reach 73 per cent at the end of the training

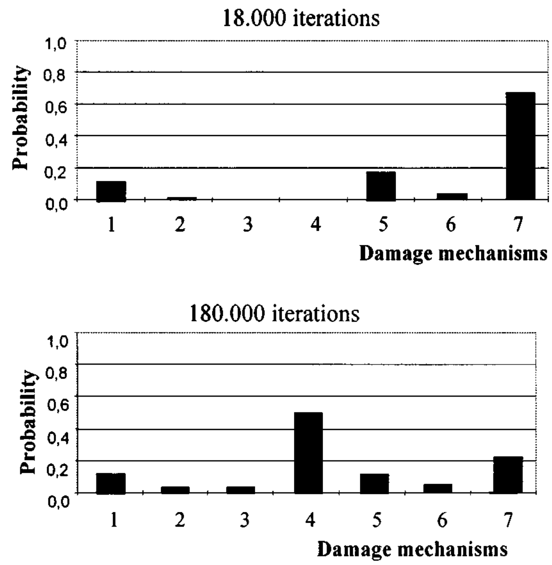

Figure 8. Prediction of damage mechanisms affecting the church of S. Maria Assunta in Scandiano

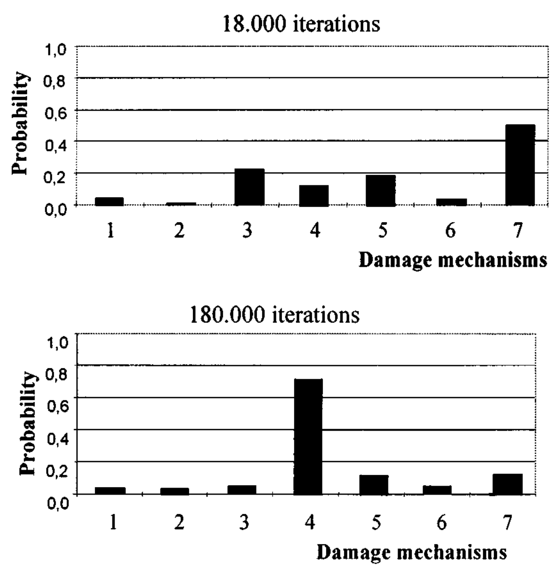

Figure 9. Prediction of damage mechanisms affecting the church of S. Chiara in Carpi

session (see Figure 9). This means that no recognition problems were experienced by the network throughout the validation stage.

6. CONCLUSIONS

Probabilistic neural networks have the advantages typical of all neural systems in the representation of functional relationships, in that they avoiding the rigidity of *a priori* mathematical models, which often do not lend themselves to the simulation of complex phenomena, while offering the transparency of probabilistic representations.

The degree of accuracy and reliability of the predictions proved better than expected. It should be noted that the predictive capability of the networks was satisfactory even when applied to samples of a mutually heterogeneous nature in terms of type of seismic event and geographical location.

The next stage of this research should focus on the acquisition of more highly refined and detailed knowledge of these issues, including the correlation between damage and seismic intensity, the effects of previous repairs and the analysis of possible correlations between damage mechanisms affering different macro-elements of the same building. At present the network is able to predict what kind of damage mechanism is most likely to occur but we expect that the information on seismic intensity will be essential in order to predict the level of activation of the damage mechanism itself. Needless to say, all this will require access to a much wider database.

The network is now in use on the database from the recent Umbria-Marche earthquake in Central Italy (September 1997). The training stage can be performed with a few more sample cases, by simply updating the network trained previously to take into account the specific characters of regional seismic actions and local building types. A first attempt to validate the network using sample cases of damaged buildings has given highly encouraging results although the analysis is not yet complete.

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